

62. What is  $(|-2x| - |3x|) - |3x| - |-5x|$  ?
- (A)  $x$     (B)  $-|x|$     (C)  $2x$     (D)  $-3|x|$     (E)  $-5|x|$
63.  $M$  is a function of  $n$  variables defined as follows:  $M(x_1, x_2, \dots, x_n) = (x_1 + x_2 + \dots + x_n)/n$ . In other words, it is simply the mean value of its arguments. Which one of the following properties is correct?
- (A)  $M(x_1, x_2, x_3, x_4) = M(M(x_1, x_2), M(x_3, x_4))$   
 (B)  $M(x_1, x_2, x_3, x_4) = M(M(x_1, x_2) + M(x_3, x_4))$   
 (C)  $M(x_1, x_2, x_3, x_4) = M(x_1 + x_2, x_3 + x_4)$   
 (D)  $M(x_1, x_2, x_3, x_4) = M(M(x_1, x_2), x_3, x_4)$   
 (E)  $M(x_1, x_2, x_3, x_4) = x_1 + M(x_2, x_3) + x_4$
64. The ellipse  $\frac{x^2}{4} + \frac{y^2}{1} = 1$  is rotated about the  $x$ -axis to form an ellipsoid. The largest possible sphere is inscribed in the ellipsoid, and then the largest possible cube is inscribed in the sphere. What is the cube's volume?
- (A)  $\frac{2\sqrt{3}}{3}$     (B)  $\frac{3}{2}$     (C)  $\frac{8\sqrt{3}}{9}$     (D)  $\frac{\pi\sqrt{3}}{3}$     (E)  $\frac{3\sqrt{3}}{2}$
65. The area of rhombus  $ABCD$  is 64 square units less than the area of square  $EFGH$  but their perimeters are equal. If  $\sin \angle AB$  is 0.05 less than  $\sin \angle EFG$ , what is the number of square units in the area of  $ABCD$  ?
66. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The process is repeated for a second cube. The two cubes are then glued together by matching the cut surfaces. Let  $a$  be the numerical value of the surface area of the resulting solid, and let  $v$  be its volume. What is  $a/v$  ?
- (A)  $4\sqrt{2}$     (B) 5    (C)  $\frac{27}{5}$     (D) 6    (E)  $\frac{27}{4}$
67. The Collatz function  $f$  is defined on the set of positive integers by the rule that if  $n$  is even then  $f(n) = n/2$ , and if  $n$  is odd then  $f(n) = 3n+1$ . Let  $f_1(n) = f(n)$ , and for  $k \geq 2$  let  $f_k(n) = f(f_{k-1}(n))$ . What is the smallest value of  $k$  such that  $f_k(9) = 1$ ?
68. A triangle has vertices at  $(0, 0)$ ,  $(2012, 0)$ , and  $(0, 40)$ . How many lattice points (points whose coordinates are both integers) lie strictly in the interior of this triangle?
69. What is the value of  $(1 + \cos \beta)^2 + \sin^2 \beta - 2 \cos \beta$  ?
- (A) 0    (B) 1    (C) 2    (D)  $\cos \beta$     (E)  $\sin^2 \beta$

70. The four consecutive vertices of rhombus  $ABCD$  have coordinates  $(a, b)$ ,  $(c, d)$ ,  $(e, f)$ , and  $(g, h)$ . If  $\log_{10} \left( \frac{b-f}{a-e} \right) = 1$  and  $d = 2 + h$ , then what is the value of  $g - c$ ?
- (A)  $-20$     (B)  $-\frac{4}{5}$     (C)  $-2$     (D)  $\frac{4}{5}$     (E)  $20$
71. In an isosceles trapezoid, the parallel bases have lengths  $\log 3$  and  $\log 192$ , and the altitude to these bases has length  $\log 16$ . The perimeter of the trapezoid can be written in the form  $\log 2^p 3^q$ , where  $p$  and  $q$  are positive integers. What is  $p + q$ ?